On Subjective Logic Trust Discount for Referral Paths

Koffi Ismael Ouattara
Huawei Technologies
Munich, Germany
koffi.ismael.ouattara@huawei.com

Ana Petrovska
Huawei Technologies
Munich, Germany
ana.petrovska@huawei.com

Artur Hermann
Ulm University, Institute of Distributed Systems
artur.hermann@uni-ulm.de

Nataša Trkulja
Ulm University, Institute of Distributed Systems
natasa.trkulja@uni-ulm.de

Theo Dimitrakos
Huawei Technologies
Munich, Germany
theo.dimitrakos@huawei.com

Frank Kargl
Ulm University, Institute of Distributed Systems
frank.kargl@uni-ulm.de

***Abstract*—Subjective Logic (SL) enriches probabilistic logic by incorporating uncertainty and subjective belief ownership, enabling the expression of uncertainty about subjective beliefs. Unlike traditional probabilistic logics, SL 1) accommodates situations where different agents express beliefs about the same proposition, integrating the subjective nature and ownership of beliefs; and 2) addresses existing limitations in Dempster-Shafer Theory of evidence (DST), particularly in modelling trust transitivity. In modern computer systems, trust assessment extends beyond direct relationships to complex networks, necessitating the consideration of referral and direct trust relationships. This paper introduces a novel trust discount operator for referral edges in complex networks, addressing challenges in discounting trust across two and multiple referral edges. Through our empirical analysis, we demonstrate the effectiveness of the proposed operator and establish a relationship between path length and trustworthiness.**

***Index Terms*—subjective logic, trust discount, referral paths, multiple agents**

I. Introduction

Subjective Logic (SL) [10], [11] is a framework for artificial reasoning, in which the general idea is to enrich probabilistic logic by explicitly including uncertainty about probabilities and subjective belief ownership. SL allows expressing a degree of (un)certainty about a subjective belief. Bayesian probability and statistics can also be employed to reason with propositions whose truth values are uncertain [8]. However, existing probabilistic logics do not capture situations where different agents express their beliefs about the same proposition [17]. SL explicitly integrates the subjective nature and ownership of beliefs in its formalism, allowing the combination of different beliefs about the same proposition. Furthermore, SL is based on the Dempster-Shafer Theory of evidence (DST), a flexible theoretical framework to represent uncertainty introduced by Dempster [4] and Shafer [20]. Concretely, DST's rule of combination, originally proposed for merging sources of evidence in DST, is also used in SL where it represents a method of preference combination embodied in SL's belief constraint operator [13]. However, the biggest drawback of DST, is that the original formalisation of DST does not model the aspect of (trust) transitivity. These limitations of the DST are addressed in the Subjective Logic theory [10], [11].

Modern computer systems mirror the complexity of our interconnected world, where autonomy, collaboration, and interdependence shape the landscape of technological progress. Namely, modern systems are becoming increasingly complex, forming compositions of components, where each component can itself be a system—also referred in the literature as system-of-systems (SoS). These subsystems collaborate, communicate, and influence one another, forming a complex whole that transcends the sum of its parts [19]. As a result, in the field of trust management and assessment, instead of assessing trust between two entities who communicate directly with each other (i. e., assessing trust on level of trust relationship), there emerges the need to assess trust on complex networks. Here the notation of referral and direct trust relationships are put at the forefront to reflect trust transitivity.

Before explaining in more detail the notion of transitivity as one of the main trust properties, we first briefly explain direct and indirect trust, as the core building block for trust transitivity. Direct trust is fundamentally based on the direct communication, direct interactions, and service behaviour between two entities. On the other hand, indirect trust is derived from 1) the recommendation passed through one or more intermediate entities or stakeholders, and 2) the existing direct interactions. If we refer to Fig. 1, then we can see that as part of SL theory, Jøsang refers to the direct trust as a functional trust, whereas to the indirect trust as a derived functional trust.

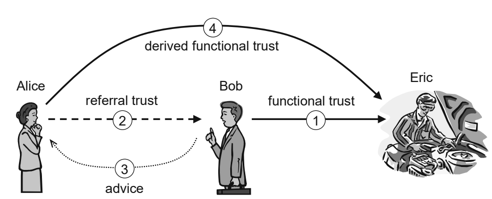

Fig. 1. Trust transitivity from [11]

Referring again to Fig. 1, according to the transitive trust principle, we can say that Alice trusts Bob that he has a good understanding in cars, and at the same time Bob trusts Eric that he is a good mechanic. As a result, Alice also trusts Eric to be able to fix her car. In this example, Alice's decision about which car mechanic to go, depends on her trust in other entities. Concretely, in this case Alice depends on her trust on Bob. Additionally, certain semantic requirements need to be satisfied for trust to be transitive. For example, every trust edge along a transitive chain must share the same trust scope. The process of obtaining the *derived functional trust* (see Fig. 1), by fusing the indirect trust (*referral trust edge*) with the direct trust (*functional trust edge*) is called trust discounting [11].

Problem. In dynamic environments, where information flows are shaped by multifaceted relationships, understanding the propagation of trust becomes crucial. However, existing methodologies often encounter challenges in discounting trust across multiple edges, particularly when confronted with referral trust opinions. Concretely, existing operators for discounting trust that have been previously proposed in [11], [12] provide only solutions for trust discounting when the final edge of the network is always a functional or direct trust. However, certain scenarios from practice, that we further elaborate in Sect. III, identify the need for discounting trust only on referral or indirect edges.

Contribution. Our main contributions are two-fold: As a first contribution, we present a novel trust discount operator for referral edges in a path. Our operator is designed to discount two referral edges in a manner that aligns with existing trust discounting principles. This advancement seeks to mitigate inconsistencies encountered when discounting multi-edge referral paths. Beyond discounting trust on only two edges, our new operator enables calculating trustworthiness of paths containing multiple referral edges within complex networks. As a second contribution, we conduct an empirical analysis. In the evaluation, we conduct a numerical analysis to support the first contribution. As part of our evaluation, we also establish a relationship between path length and trustworthiness. Our findings highlight that longer referral paths tend to yield less trustworthy outcomes.

In summary, this paper not only addresses existing challenges in trust discounting but also advances the frontier by providing robust methodologies for evaluating and quantifying trust within complex networks.

Structure. The rest of this paper is structured as follows: Sect. II presents the related work. In Sect. III, we provide a use case instantiation used as a running example throughout the rest of the paper. In Sect. IV, we present the background on SL that is relevant for our contribution, before discussing in detail of the problem statement in Sect. V. The newly proposed Referral-Edge Path Discounting Operator is described in Sect. VI. In Sect. VII, we conduct empirical evaluation to show the effectiveness of our solution. Finally, Sect. VIII concludes the paper.

II. Related work

Information and sensor fusion. In various domains, such as sensor fusion, expert system development, and most notably multi-agent systems, there is an increasing demand for information fusion or aggregation. Within the realm of multi-agent systems, knowledge refers to the information acquired through agents' observations, often termed as belief or belief base [18]. Previous research efforts [9], [18] have represented belief using propositional logic-based formalisms. For instance, Grégoire and Konieczny [9] conducted a survey on logic-based information fusion approaches, exploring their connection to multi-agent negotiation. Lastly, there have been various methods for belief fusion proposed in the literature [15], [18], [21], [23] that provide general basis for knowledge or information aggregation. However, the above-mentioned studies have primarily focused on the field of information systems.

Munz and Dietmayer [16] propose an approach to enhance sensor fusion system detection performance, measured in terms of detection rate versus false alarm rate. To achieve this, they employ an algorithm that directly incorporates Dempster-Shafer Theory-based sensory information. In other related work [22], Dempster-Shafer Theory (DST) is also utilized to model sources of uncertainty before applying evidence fusion. [16], [22] are mainly concerned with the fusion of data in a single system, where the set of sensors are the multi-sources, rather than the fusion of information or knowledge across multiple independent systems in a multi-agent system setup.

Mathematical models for reasoning and information fusion. When we assume an objective world, we can use binary logic to assert propositions about a state of the world to be either true or false [3]. However, the world is unpredictable, and in many situations, one cannot determine the nature of a proposition with certainty. Through probability calculus, which takes argument probabilities in the range [0,1], we allow propositions to be partially true. However, due to the lack of sufficient evidence, we are often unable to estimate probabilities with confidence. Also, whenever the truth of a proposition is assessed, it is always done by an individual, and it cannot be considered to represent a general and objective belief.

To reason with propositions whose truth-values are uncertain, Bayesian probability and statistics can also be employed [7]. However, this type of probabilistic logic does not allow to seamlessly model situations where different agents express their beliefs about the same proposition. Dempster-Shafer Theory (DST) [2], [20] and Subjective Logic (SL) [10], [11] explicitly integrate the subjective nature and ownership of beliefs in its formalism, allowing the combination of different beliefs about the same proposition.

In DST, probability values are assigned to sets of possibilities rather than single events. Furthermore, Dempster-Shafer's rule of combination is associative, commutative and non-idempotent [13] and the functions are simple to implement and compute. These properties of Dempster-Shafer's rule of combination are beneficial for real-time applications as sources can be combined sequentially, and at a random order [6]. However, these results should be considered with caution, as a later study [6] showed that the order in which sources get aggregated may impact the results. Another downside of the DST is that it can lead to counter-intuitive results when combining conflicting sources [24], meaning that the rule cannot be applied if the two sources are in complete opposition. Finally, the original formalisation of DST does not model the aspect of trust transitivity. These limitations of the DST are addressed in the Subjective Logic (SL) theory [10], [11]. In Sect. IV, we explain in more detail the relevant concepts from SL, that are relevant for our contributions.

Trust Propagation and Transitivity. Jøsang et al. in [12] investigates possible formalism for different types of trust propagation and transitivity implemented using belief reasoning based on SL. As part of this work the authors propose three trust transitivity operators: *Uncertainty Favouring Trust Transitivity*, *Opposite Belief Favouring* and *Base Rate Sensitive Transitivity*. However, the limitation of these three operators is the same as the fusion operators that are originally proposed in the SL [11]. Namely, none of the existing operators supports trust discounting only on referral edges.

III. Running Example

To motivate the need for the contributions of this paper, we introduce a reference problem from the automotive domain, in particular Intersection Management Assist (IMA) application, which is also used as a running example throughout the paper.

IMA is considered to be a critical component in Cooperative Intelligent Transportation Systems (C-ITS)—applications where Intelligent Transport Systems (e. g., vehicles, infrastructure equipment, traffic control centres) communicate and share information in order to improve road safety, traffic efficiency, sustainability, etc. The Car2Car consortium [1] makes a distinction between various stages (from Day 1 to Day 3 applications). Several of the Day 2 and Day 3 applications are mainly intended for automated vehicles, in which case C-ITS starts to converge into Connected and Cooperative Automated Mobility (CCAM).

In order to reflect the dynamic nature and heterogeneity of C-ITS applications and the dynamic environments in which the systems operate, no initial trust between systems should be assumed. Instead, trust needs to be built from the ground up based on relevant evidence, and also needs to be continuously re-evaluated. Accessing trust in these situations is paramount since the vehicles need to establish a sufficient level of trust in one another to collaboratively execute safety-critical tasks, e. g., to autonomously cross an intersection.

Let us take as an example the following IMA intersection scenario, shown in Fig. 2. As part of the IMA scenario, vehicle A communicates with vehicles B and C through a vehicle-to-vehicle (V2V) communication. V2V enables vehicles to transmit data and communicate with one another and share information about their driving behaviours [5]. On the other hand, vehicles B and C can communicate with vehicle D that is approaching the intersection. In this scenario, vehicle A is interested about the velocity of vehicle D, v_D. Although vehicle A cannot directly communicate with vehicle D; and therefore, cannot directly assess the trustworthiness of v_D, it can build the level of trust on v_D indirectly, through B and C.

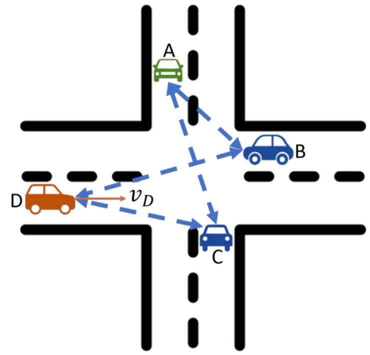

Fig. 2. Example of IMA scenario

In this section, we consider a use case from the automotive domain; however, the model problem of this paper is relevant for various other domains. For example, cyber-physical systems (CPSs), IoT, networking and 6G, data security, etc. We elaborate the model problem in more details in Sect. V.

IV. Background on Subjective Logic

Subjective Logic (SL) [10], [11] is a framework for artificial reasoning based on probabilistic logic and Dempster-Shafer theory of evidence [4], [20]. SL inherently allows:

1) uncertainties representation as part of the fundamental building block of SL, called *subjective opinion*, and
2) reasoning about the uncertainties through a process of *belief fusion* in which multiple subjective opinions are aggregated based on the selected fusion operator.

We explain subjective opinions and belief fusion in more detail in Sect. IV-A. Additionally, in Section IV-B, we explain more advanced SL concepts—*subjective trust networks* and *trust discounting*—since they are relevant for the contributions that we make as part of this paper.

A. Subjective Opinions and Belief Fusion

1) Subjective Opinions: are the fundamental building blocks of SL. The representation of a subjective opinion is a composite function consisting of belief masses, uncertainty mass and base rate. Formally, a subjective opinion expresses a belief about a state of a variable X which takes its values from a domain $\mathbb{X}$ (i. e., a state space). The domain is the set of all different possible states for X. The different values of a domain are exclusive, i. e., only one state value is possible at any moment in time-, and exhaustive, i. e., all possible state values are included in the domain. Domains can be binary (exactly two values) or n-ary (n values) where $n > 2$. A binary domain can be denoted $\mathbb{X} = \{x, \overline{x}\}$, where $\overline{x}$ is the complement (negation) of x.

The notation ω_X^A is used to denote opinions in subjective logic, where the subscript X indicates the target variable or proposition to which the opinion applies, and the superscript A indicates the subject agent/source who holds the opinion.

Definition 1 (Binomial Opinion [11]). Let $\mathbb{X} = \{x, \overline{x}\}$ be a binary domain with binomial random variable $X \in \mathbb{X}$. A binomial opinion about the truth/presence of value x is the ordered quadruplet $\omega_x = (b_x, d_x, u_x, a_x)$, where the additivity requirement $b_x + d_x + u_x = 1$ is satisfied, and where the respective parameters are defined as:

b_x : belief mass in support of x being TRUE (i.e. $X = x$),

d_x : disbelief mass in support of x being FALSE (i.e. $X = \overline{x}$),

u_x: uncertainty mass representing the vacuity of evidence,

a_x : base rate, i.e., prior probability of x without any evidence.

The projected probability of a binomial opinion about value x is defined by:

$$P(x) = b_x + a_x u_x. \tag{1}$$

2) Belief Fusion: allows multiple opinions regarding the same proposition to be merged or aggregated into a single, collective opinion. Several *fusion operators* have been proposed that are unique to the theory: *constraint fusion, cumulative fusion, averaging fusion, weighted fusion*, and *consensus & compromise fusion*. Depending on the concrete scenario, we might need to choose among different fusion operators with different properties. Additionally, Jøsang [11] has also proposed a *trust discount operator* for deriving trust on transitive paths. Since the trust discounting operator is essential for our work, we explain it in more detail in the upcoming section. For a more detailed explanation on the other operators, including their mathematical definitions, please refer to [11], [14].

B. Subjective Trust Networks and Trust Discounting

Subjective Logic can be used to analyse trust through Subjective Trust Networks (STNs). STNs represent trust and belief relationships from agents, via other agents and sensors to target entities/variables, where each trust and belief relationship is expressed as a subjective opinion (ω) [11]. In Fig. 3, we show a simple example of an STN graph, where the goal is to derive the functional trust ω^A_X. The main SL operators used for STN graph processing are the *fusion operators* and *trust discounting*. For example, ω^A_X is calculated as follows:

$$\omega^A_X = \omega^{[A;B]}_X \odot \omega^{[A;C]}_X = (\omega^A_B \otimes_{TE} \omega^B_X) \odot (\omega^A_C \otimes_{TE} \omega^C_X), \tag{2}$$

where the discounted opinions $\omega^{[A;B]}_X = \omega^A_B \otimes_{TE} \omega^B_X$ and $\omega^{[A;C]}_X = \omega^A_C \otimes_{TE} \omega^C_X$ are fused together.

Previously, we explained that there are various types of fusion operators and the decision on which operator should be used is application- and use case-dependent. Hence, in Eq. (2), with $\odot$ we annotate a generic fusion operator.

Referring again to one of the paths in Fig. 3 (e. g., ABX and Fig. 4), we can see that analyst A receives information from B, which could be a vehicle or a sensor in an example from the automotive domain. Analyst A forms an opinion on the trustworthiness of B regarding the specific information

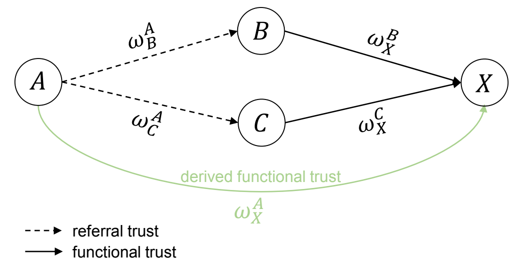

Fig. 3. STN example

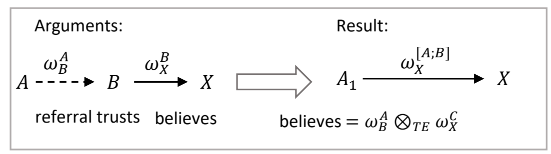

Fig. 4. Trust discounting for Two-Edge Path [11, Ch.14.3]

(trust scope) provided about a variable X. According to the transitive trust principle, the figure shows that A trusts B (though referral trust), since A knows that B has a direct observation or experience with X for a concrete scope (i. e., B has a functional or direct trust on X). Using both A's trust in B (ω^A_B) and B's opinion about X (ω^B_X) received by A, A derives an opinion about X (ω^A_X).

C. Different Trust Discounting Operators

In his book [11], Jøsang proposes two trust discount operators for 1) Two-Edge Path and 2) Multi-Edge Path. The process of trust discounting for Two-Edge Path is shown in Fig. 4. As the name suggests, this trust discounting operator ($\otimes_{TE}$) discounts the opinions on two edges: one referral trust ω^A_B from node A to node B, and one functional trust ω^B_X from node B to variable X.

The Two-Edge Path Discounting Operator ($\otimes_{TE}$) is defined as follows [11]:

$$\omega^{[A;B]}_X = \omega^A_B \otimes_{TE} \omega^B_X : \begin{cases} b^{[A;B]}_X & = P^A_B b^B_X \\ d^{[A;B]}_X & = P^A_B d^B_X \\ u^{[A;B]}_X & = 1 - (b^{[A;B]}_X + d^{[A;B]}_X) \\ a^{[A;B]}_X & = a^B_X \end{cases} \tag{3}$$

where P^A_B is the projected probability (see Eq. (1)). The Two-Edge Path discount operator is equivalent to the *Base Rate Sensitive Transitivity* that Jøsang et al. propose in [12] (cf. Sect. II).

The trust discounting for Multi-Edge Path describes how trust discounting is performed for longer trust paths, in situations when there are more than two adjacent edges, as shown in Fig. 5. Namely, Fig. 5 shows a trust path from node A_1 to variable X via an arbitrary number of intermediate nodes $A_2, A_3, \ldots, A_n$.

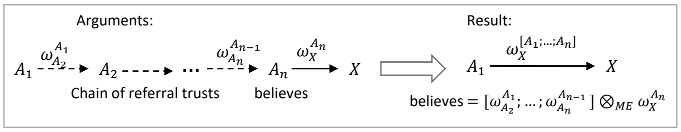

Fig. 5. Trust Discounting for Multi-Edge Path [11, Ch.14.3]

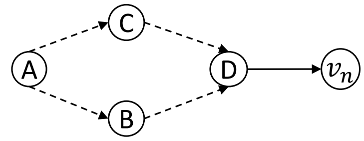

Fig. 6. STN for the IMA scenario

The derived opinion $\omega_X^{[A_1;\ldots;A_n]}$ on Multi-Edge Paths is defined as follows:

$$\omega_X^{[A_1;\ldots;A_n]} = [\omega_{A_2}^{A_1};\ldots;\omega_{A_n}^{A_{n-1}}] \otimes_{ME} \omega_X^{A_n} : \tag{4}$$
$$\begin{cases} b_X^{[A_1;\ldots;A_n]} & = P_{A_n}^{A_1} b_X^{A_n} \\ d_X^{[A_1;\ldots;A_n]} & = P_{A_n}^{A_1} d_X^{A_n} \\ u_X^{[A_1;\ldots;A_n]} & = 1 - (b_X^{[A_1;\ldots;A_n]} + d_X^{[A_1;\ldots;A_n]}) \\ a_X^{[A_1;\ldots;A_n]} & = a_X^{A_n} \end{cases}$$

where $P_{A_n}^{A_1}$ is the projected probability of the referral trust path $[A_1;...;A_n]$, computed as

$$P_{A_n}^{A_1} = \prod_{i=1}^{n} P_{A_{i+1}}^{A_i}. \tag{5}$$

We want to emphasise that the operators for Two-Edge Path Trust Discounting ($\otimes_{TE}$) and Multi-Edge Path Trust Discounting ($\otimes_{ME}$) are two different operators. Hence, we label them with two different symbols, as we also show in Tab. I. Additionally, please note that in order to apply these discount operators, the last edge should always be a functional trust edge.

TABLE I
TRUST DISCOUNT OPERATORS

Operator Name	Symbol	Function
Two-Edge Path Eq. (3), Fig. 4	$\otimes_{TE}$	$\omega_B^A \otimes_{TE} \omega_X^B$
Multi-Edge Path Eq. (4), Fig. 5	$\otimes_{ME}$	$[\omega_{A_2}^{A_1};\ldots;\omega_{A_n}^{A^{n-1}}] \otimes_{ME} \omega_X^{A_n}$
Referral-Edge Path Eq. (6), Fig. 7	$\otimes_{RE}$	$((\omega_{A_2}^{A_1} \otimes_{RE} \ldots) \otimes_{RE} \omega_{A_{n-1}}^{A_{n-2}}) \otimes_{RE} \omega_{A_n}^{A_{n-1}}$

V. Problem Statement

Based on the IMA scenario from the running example in Sect. III, a trust model (i. e., an STN) can be derived, as shown in Fig. 6. As part of the STN in the concrete running example, the four vehicles (A, B, C and D) and the data items (precisely, Vel_D) are represented as trust objects or nodes in the graph. The STN has four referral trust relationships or edges (ω_B^A, ω_D^B, ω_C^A, and ω_D^C) forming a chain, as can be seen in the figure. Additionally, vehicle D has a direct or a functional trust on its velocity Vel_D. Also, as previously explained in Sect. III, the final goal is to calculate the opinion $\omega_{Vel_D}^A$ that the vehicle A has on the velocity Vel_D even though the vehicle does not observe the velocity directly (i.e., it does not have a direct relationship with the velocity). This is done by first discounting the referral trust opinions ACD and ABD, and then fusing the obtained opinions.

As previously explained in section IV-B and shown in Eq. (3) and (4), the existing trust discounting operators need the last edge to be a functional trust, which is not the case in ACD and ABD paths, where CD and BD are also referral trust. Although in the previous work Jøsang [11] differentiates and proposes trust discounting operators for Two-Edge and Multi-Edge paths, the case where the graph topology consists *only* of referral chains, has been left out of consideration in his theoretical contributions. To overcome this problem, one might be tempted to use the existing trust discounting operator for Two-Edge Path (Eq. (3)) to discount AC and CD. However, this operator was not built for this purpose, primarily because the second opinion, in this case CD is expected to be a functional trust edge. Furthermore, if we want to discount two referral trust edges as part of a chain topology, as shown in Fig. 6, the solution needs to be consistent with the trust discounting operator for Multi-Edge Path, which already considers discounting on multiple referral edges. Another solution could be to discount using only projected probability since this is the exact way that the Multi-Edge Path uses to discount referral edges (cf. $P_{A_n}^{A_1}$ in Eq. (4)). This means that only the projected probabilities (see Eq. (5)) are used instead of the opinions; therefore, resulting in a projected probability again. However, in our example in Fig. 6, the node D requires fusing of the two previously discounted referral paths ACD and ABD, which is only possible when we have opinions (not probabilities).

In this work we aim to build a trust discounting operator for two or more referral trust opinions (without having the last edge in the STN as a functional trust), since this is needed for some STN topology types, as we showed in our running example and Fig. 6. Moreover, the new trust discounting operator for two referral trust opinions must provide consistent results with the already existing operators that Jøsang introduced for Two-Edge Path and Multi-Edge Path trust discounting.

VI. The Proposed Referral-Edge Path Discounting Operator

As part of this section, we propose a new Referral-Edge Path Discounting Operator ($\otimes_{RE}$) to address the problem that we previously explained in Sect. V. Concretely, as it can be seen in Fig. 7, in this paper we propose a new operator that enables discounting on two or more referral edges. Compared to the other trust discounting operators, that we explained in detail in Sect. IV-C, our newly proposed operator addresses the issue of not having a functional trust at the end of the path.

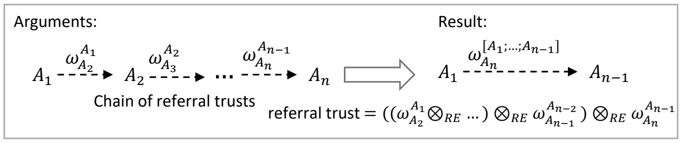

Fig. 7. Trust Discounting for Referral-Edge Path

The idea behind the new Referral-Edge Path Discounting Operator is derived from the principles of the Uncertainty Favouring Operator that was previously proposed in [12]. A notable distinction between our operator (see Eq. (6)) and the uncertainty favouring operator is that we focus on ensuring the projected probability of the resulting discounted opinion (e. g., $P_C^{[A;B]}$) to be equal to the product of the projected probabilities of the two original opinions prior discounting (e. g., $P_C^{[A;B]} = P_B^A P_C^B$). We prove this in theorem 2. There are two ways to achieve this, either by adjusting the belief and the disbelief, or the base rate. We decided to adjust the base rate because the symmetry between belief and disbelief implies that altering one necessitates a corresponding adjustment to the other. Furthermore, if we refer to Fig. 4, when discounting the referral trust edge AB and functional trust edge BX, we can observe that the base rate of the final discounted opinion $\omega_X^{[A;B]}$ is equal to the base rate of the functional trust edge. Therefore, adjusting the base rate of the referral trust will not impact the base rate of the ultimate derived opinion.

We define the Referral-Edge Discounting Path Operator ($\otimes_{RE}$) as follows:

$$\omega_C^{[A;B]} = \omega_B^A \otimes_{RE} \omega_C^B : \qquad (6)$$

$$\begin{cases} b_C^{[A;B]} &= b_B^A b_C^B \\ d_C^{[A;B]} &= b_B^A d_C^B \\ u_C^{[A;B]} &= 1 - (b_C^{[A;B]} + d_C^{[A;B]}) \\ a_C^{[A;B]} &= \frac{(b_B^A + u_B^A a_B^A)(b_C^B + u_C^B a_C^B) - b_B^A b_C^B}{1 - b_B^A (b_C^B + d_C^B)} \end{cases}$$

Our proposed operator works in the same way for discounting two or multiple referral edges (see Fig. 7) without any inconsistency as we show in the following theorem 1. This also states that the operator is associative.

Theorem 1. *The Operator $\otimes_{RE}$ is associative. In other words, for all referral paths consisting of three edges AB, BC, and CD the following holds:*

$$(\omega_B^A \otimes_{RE} \omega_C^B) \otimes_{RE} \omega_D^C = \omega_B^A \otimes_{RE} (\omega_C^B \otimes_{RE} \omega_D^C). \qquad (7)$$

Before proving theorem 1 we state the second relevant theorem as follows:

Theorem 2. *The projected probability of the resulting discounted opinion obtained after applying the proposed operator ($\otimes_{RE}$) defined in Equation 6 is equal to the product of the projected probabilities of the two original opinions prior discounting. In other words, $P_C^{[A;B]} = P_B^A P_C^B$.*

Due to space limitation, the proof of theorem 2 is out of the scope of this paper.

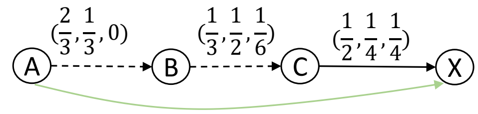

Fig. 8. Three edges path with numerical value

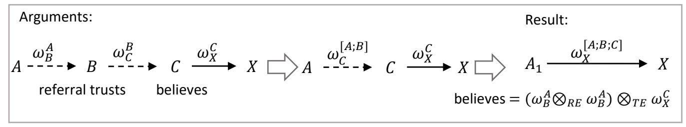

Fig. 9. Method 2 applied to STN depicted in Fig. 8

Proof 1. The belief of $(\omega_B^A \otimes_{RE} \omega_C^B)$ is equal to $b_B^A b_C^B$ (cf. $b_C^{[A;B]}$ in (6)). Therefore, the belief of $(\omega_B^A \otimes_{RE} \omega_C^B) \otimes_{RE} \omega_D^C$ is equal to $b_B^A b_C^B b_D^C$ and its disbelief is equal to $b_B^A b_C^B d_D^C$. According to theorem 2, we know that its projected probability is $P_B^A P_C^B P_D^C$. On the other hand, the belief of $(\omega_C^B \otimes_{RE} \omega_D^C)$ is equal to $b_C^B b_D^C$ and its disbelief is equal to $b_C^B d_D^C$. Therefore, the belief of $\omega_B^A \otimes_{RE} (\omega_C^B \otimes_{RE} \omega_D^C)$ is equal to $b_B^A b_C^B b_D^C$ and its disbelief is equal to $b_B^A b_C^B d_D^C$. Same as previously, its projected probability is $P_B^A P_C^B P_D^C$.
Since both quantities have the same belief and disbelief, we can deduct that they have the same uncertainty. Then they have the same belief, disbelief, uncertainty and projected probability. Therefore, we can conclude that they have same base rate which means they are equal. $\square$

Note that $\otimes_{RE}$ is non-idempotent. We can prove that for an opinion $\omega = (b, d, u, a)$, $\omega \otimes_{RE} \omega = \omega \Leftrightarrow b = 1$ or $u = a = 1$

A. Operators equivalence

To show the consistency between our newly proposed Referral-Edge Path discounting operator and Multi-Edge Path discounting operator proposed in [11], we reduce the STN depicted in Fig. 8. By reducing we refer to ω_X^A opinion derivation. We can distinguish two methods for this purpose.

1) Method 1—Trust Discounting with Multi-Edge Path: In this method we use the Multi-Edge Trust Discounting operator (Eq. 4 and Fig. 5) to compute ω_X^A through the transitive path $ABCX$. This computation is done by firstly calculating P_C^A of the referral path ABC which is set to $P_B^A P_C^B$. Then multiply P_C^A by the belief and disbelief of the functional trust ω_X^C as follows:

$$\omega_X^{[A;B;C]} : \begin{cases} b_X^{[A;B;C]} &= P_C^A b_X^C \\ d_X^{[A;B;C]} &= P_C^A d_X^C \\ u_X^{[A;B;C]} &= 1 - (b_X^{[A;B;C]} + d_X^{[A;B;C]}) \\ a_X^{[A;B;C]} &= a_X^C \end{cases} \qquad (8)$$

where $P_C^A = P_B^A P_C^B$.

2) Method 2—Trust Discounting with Referral-Edge Path (our new discounting operator) + Two-Edge Path: Here we firstly discount the two referral trust opinions (ω_B^A, ω_C^B) using our new operator (Eq. (6)) to calculate $\omega_C^{[A;B]}$. Then we discount $\omega_C^{[A;B]}$ with ω_X^C using the Two-Edge Discount ($\otimes_{TE}$) calculated as follows:

$$\omega_X^{[A;B;C]} : \begin{cases} b_X^{[A;B;C]} & = P_C^{[A;B]} b_X^C \\ d_X^{[A;B;C]} & = P_C^{[A;B]} d_X^C \\ u_X^{[A;B;C]} & = 1 - (b_X^{[A;B;C]} + d_X^{[A;B;C]}) \\ a_X^{[A;B;C]} & = a_X^C \end{cases} \tag{9}$$

As depicted in Fig. 9, this method is a step by step process, which is not the case for Method 1.

To show equivalence between these two methods, let's denote by ω_1 and ω_2 the result obtained using Method 1 and Method 2, respectively. Our goal is to prove that $\omega_1 = \omega_2$ using theorem 2:

$$\begin{aligned} P_C^{[A;B]} = P_B^A P_C^B &\Leftrightarrow P_C^{[A;B]} = P_B^A P_C^B \\ &\Leftrightarrow P_C^{[A;B]} = P_C^A \\ &\Leftrightarrow \omega_1 = \omega_2. \end{aligned}$$

VII. Evaluation

A. Comparing our newly proposed operator with Two-Edge and Multi-Edge Path

As part of Sect. VI-A, we have shown the equivalence of the two methods, one based on Multi-Edge Path and the second one based on Referral-Edge Path+Two-Edge Path, analytically. In the following, we make numerical evaluations based on the following experimental setups, on the example of the STN shown in Fig. 8:

Setup 1 Calculating ω_X^A using Multi-Edge Path (Method 1)
Setup 2 Calculating ω_X^A using Referral-Edge Path + Two-Edge Path (Method 2)
Setup 3 Calculating ω_X^A using two times Two-Edge Path.

First, using Setup 1 and 2, we provide a numerical evaluation of Method 1 and Method 2. Using the opinion values from Fig. 8, we calculate the following:

1) Discounted opinion according to Setup 1: $\omega_X^A = [\omega_B^A; \omega_C^B] \otimes_{ME} \omega_X^C = (0.14, 0.07, 0.79)$
2) Discounted opinion according to Setup 2: $\omega_X^A = (\omega_B^A \otimes_{RE} \omega_C^B) \otimes_{TE} \omega_X^C = (0.14, 0.07, 0.79)$

As we can observe, applying methods 1 and 2 yields the same result.

Second, in Sect. V we elaborated that the Two-Edge Path operator cannot be used to discount two referral edges. To support this statement, we want to show the inconsistency when using the Two-Edge Path operator to discount two referral edges (Setup 3). Using again Fig. 8, we calculate the following discounted opinion: $\omega_X^A = (\omega_B^A \otimes_{TE} \omega_C^B) \otimes_{TE} \omega_X^C = (0.22, 0.11, 0.67)$. As we can see from the calculated discounted opinions in Setup 1 and Setup 3, they result in different numerical opinions. With this we show that the Two-Edge Path operator cannot be used in the same way as our newly proposed operator.

B. Evaluating referral paths

In Fig. 10 we show an example of a generic path, that could be part of an STN. As you can see in the figure, it only contains referral edges with a varying number of intermediate nodes (i. e., a varying number of edges). Please note that each edge has the same numerical opinion. Based on this path, in Fig. 11

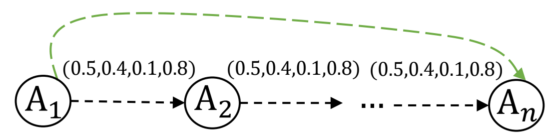

Fig. 10. Generic referral path with the same numerical opinion for each edge

we show the evolution of belief, disbelief, uncertainty and projected probability when increasing the number of referral edges in the path. In each plot in Fig. 11, we differentiate between discounting only with Referral-Edge Path operator (RE, shown in orange) using Eq. (10) versus discounting only with Two-Edge Path operator (TE, shown in blue) using Eq. (11).

$$\begin{aligned} \omega_{A_n}^{A_1} &= (\omega_{A_2}^{A_1} (\otimes_{RE} \cdots \otimes_{RE} (\omega_{A_{n-1}}^{A_{n-2}} \otimes_{TE} \omega_{A_n}^{A_{n-1}}) \\ &= ((\omega_{A_2}^{A_1} \otimes_{RE} \dots) \otimes_{RE} \omega_{A_{n-1})}^{A_{n-2}} \otimes_{RE} \omega_{A_n}^{A_{n-1}} \end{aligned} \tag{10}$$

$$\begin{aligned} \omega_{A_n}^{A_1} &= \omega_{A_{n-1}}^{[A_1;\dots;A_{n-2}]} \otimes_{ME} \omega_{A_n}^{A_{n-1}} \\ &= (\omega_{A_2}^{A_1} \otimes_{TE} (\cdots \otimes_{TE} (\omega_{A_{n-1}}^{A_{n-2}} \otimes_{TE} \omega_{A_n}^{A_{n-1}}) \\ &\neq ((\omega_{A_2}^{A_1} \otimes_{TE} \dots) \otimes_{TE} \omega_{A_{n-1})}^{A_{n-2}} \otimes_{TE} \omega_{A_n}^{A_{n-1}} \end{aligned} \tag{11}$$

From Eq. (11), we can observe the following. First, the equation shows that Two-Edge Path operator is not associative, whereas our operator is (as we also proved before). Second, if we start the discounting process from the last edges, then the result is equal to discounting with Multi-Edge Path Operator. For that reason in Fig. 11 we only compare between Two-Edge Path and Referral-Edge Path operators.

In Fig. 11, as the number of edges increases, for both TE and RE, the belief and the disbelief decrease, whereas the uncertainty increases. When it comes to the projected probability, for RE it decreases until reaching 0, whereas for TE it increases until it reaches 0.8 which is the base rate.

In general for all discount operators (Eq. (3), (4), (6)), when one calculates the discounted opinion $\omega_C^{[A;B]}$ (referring to the referral path ABC in Fig. 8), the uncertainty mass obtained is bigger than the uncertainty mass of the original opinion ω_C^B. How much it grows depends on the operator. According to Eq. (3) and (6), the belief b of the resulting opinion from the operator $\otimes_{TE}$ ($b_B^A = P_B^A b_C^B$) is bigger than the belief of the resulting opinion from $\otimes_{RE}$ ($b_B^A = b_B^A b_C^B$), since the projected probability is $p = b + au \geq b$. Furthermore, the disbelief d of the resulting opinion from $\otimes_{TE}$ ($d_B^A = P_B^A d_C^B$) is bigger than the disbelief of the resulting opinion from $\otimes_{RE}$ ($d_B^A = b_B^A d_C^B$). Since the uncertainty is calculated with $u = 1 - (b + d)$, the uncertainty of the resulting opinion from $\otimes_{TE}$ is smaller than

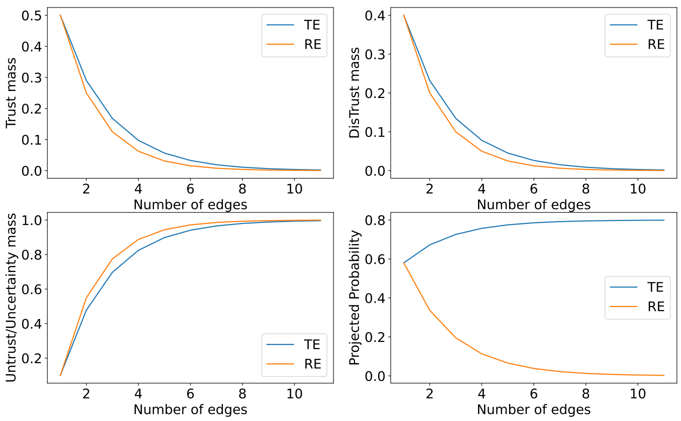

Fig. 11. Comparison of discounting referral edges using Two-Edge Path operator (TE) and the new operator, Referral-Edge Path (RE) on a transitive graph consisting in a varying number of referral edges with all edges having the same trust opinion: (0.5, 0.4, 0.1, 0.8) as depicted in Fig. 10.

the uncertainty of the resulting opinion from $\otimes_{RE}$. As shown in the uncertainty plot in Fig. 11, the curve for RE is above TE.

Even though the Referral-Edge Path operator increases the uncertainty compared to the Two-Edge Path, it ensures that the projected probability of the discounted opinion is not as strongly impacted by the base rate of each opinion.. This is not the case for the TE operator, where the projected probability tends to the base rate of the opinion (0.8), as we can see in the bottom right plot. Indeed, the base rate of the final derived opinion is set to the base rate of the last opinion (cf. Eq. (3)) which is equal to 0.8. Thus, since the uncertainty tends to 1 and belief/disbelief tends to 0, we get for the longer paths (approximately for the paths with more than six edges) the probability $p = b + au \approx a = 0.8$. The new operator allows to get a discounted opinion for which the projected probability is not too much sensitive to the base rate. This could therefore help to distinguish which referral path is the more trustworthy than the other, which is overwritten by the importance of the base rate in the Two-Edge Path operator. Moreover we observe that the longer the path, the smaller the projected probability is, which is in alignment with the referral paths $[\omega_{A_2}^{A_1}; \ldots; \omega_{A_n}^{A^{n-1}}]$ of the Eq. (4) for Multi-Edge Path operator. This should be expected since having several nodes in a transitive path should result in a more untrustworthy path, no matter if the base rate is high or low.

VIII. Conclusion

In summary, our paper presents a significant contribution to the field of trust management within complex networks. By introducing the Referral-Edge Path Discounting Operator, we address existing challenges in trust evaluation, particularly in dynamic environments. Through empirical analysis, we validate the efficacy of our solution and establish its relevance across various domains including automotive systems, cyber-physical systems, and data security. This work not only enhances our understanding of trust propagation but also provides robust methodologies for quantifying trust, thereby advancing the frontier of trust assessment in modern systems.

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